

PHILOSOPHICAL FOUNDATIONS OF QUALITATIVE SPATIAL REASONING

A CASE FOR BERTRAND RUSSELL

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- ▶ Bertrand Russell wrote only two major works where geometry plays an important role: *An Essay on The Foundations of Geometry* (EFG, 1897) and *Philosophy of Mathematics* (PoM, 1903)
- ▶ His contribution in that respect is, for the most part, forgotten
- ▶ These two works are different in many aspects: what is common to them is the focus on a special type of geometry
- ▶ In fact, this geometry can be seen to underpin all his major philosophical works!



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- ▶ In both these works Russell emphasises the importance of what he calls **QUALITATIVE GEOMETRY**
- ▶ However, a closer look reveals that there are important differences in the way he uses this and related terms
- ▶ The untangling of these is the main focus of this presentation
- ▶ I will also say why this is important for a contemporary theoretical computer scientist



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RUSSELL ON QUALITATIVE GEOMETRY

“... qualitative *or* descriptive ...” (EFG, p.9)

descriptive geometry is to be established as a priori
(EFG, p. 143)

Euclid does not have too many descriptive theorems,
only metrical (EFG, p. 118)

“measurement is no longer regarded as fundamental
and space is dealt with by descriptive rather than quan-
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PROJECTIVE GEOMETRY

*logical independence of **projective geometry** can only be maintained by a purely descriptive definition (EFG, p. 122)*

*“Projective Geometry deals only with quality — **for which reason it is called descriptive** — and cannot distinguish between two figures which are qualitatively alike” (EFG, p. 33)*

“quantities, as used in PG, do not stand for spatial magnitudes but are conventional symbols for purely qualitative spatial relations” (EFG, p. 36)



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ORIGINS

'In Geometry, von Staudt's quadrilateral construction and Pieri's work on Projective Geometry have shown how to give points, lines and planes an order independent of metrical considerations of quality, while Descriptive Geometry proves that a very large part of Geometry demands only the possibility of serial arrangement. Moreover the whole philosophy of space and time depends upon the view we take of order' (PoM, p. 201)



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DESCRIPTIVE VS. PROJECTIVE

“The subject which I have called Descriptive Geometry is not, as a rule, sharply distinguished from Projective Geometry” (PoM, p. 399)

“The distinction between Projective and Descriptive geometry is very recent and is of essentially ordinal nature” (PoM, p. 425)



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AFFINE GEOMETRY

*“...these differences are not very important for our present purpose and it will therefore be convenient to speak of projective and descriptive Geometry together as **non-quantitative** Geometry.” (PoM, p 425)*

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RUSSELL'S RELEVANCE TODAY

- ▶ Speaking in modern terms, I claim that both projective and affine geometry informed Russell's philosophical ideas
- ▶ This can be seen from *The Problems of Philosophy* (1912), to *On Our Knowledge of the External World* (1914) even to *Human Knowledge* (1948)
- ▶ Because of the emphasis he put on the notion of **quality** in geometry, Bertrand Russell should be considered one of the forefathers of **Qualitative Spatial Reasoning** paradigm, a subfield of Knowledge Representation and Reasoning.



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Thank you.

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