Meaning as Algorithm

Ivo Pezlar

Czech Academy of Sciences, Institute of Philosophy
Prague, Czech Republic

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Introduction and motivation

Main goal

- brief history of algorithmic (procedural, ...) theories of meaning (ATM) for natural language
  - from logical and philosophical point of view

Outline

Forerunners

Gottlob Frege (1848–1925)

- 'meaning as a mode of presentation' [Frege, 1879]
  - 'way of being given' ('Art des Gegebenseins')
  - mode of presentation of denotation = procedure for determining denotation
  - e.g., $5+7=12$ and $6+6=12$, same truth values, different methods of reaching them

> It is natural, now, to think of there being connected with a sign (name, combination of words, written mark), besides that which the sign designates, which may be called the meaning of the sign, also what I should like to call the *sense* of the sign, wherein the mode of presentation is contained. (p. 158, orig. p. 26)

Forerunners

Ludwig Wittgenstein (1889–1951)

- 'the meaning of a word is its use in the language(-game)' ([Wittgenstein, 1953], I, sec. 43)
- procedural approach: understanding an expression = knowing how the expression is used = knowing how to 'execute' the meaning
- meaning is a kind of 'doing'
  - offshoot: conceptual role semantics, inferentialism, proof-theoretic semantics

> I shall also call the whole, consisting of language and the actions into which it is woven, the "language-game". (I, sec. 7)

> Here the term "language-*game*" is meant to bring into prominence the fact that the *speaking* of language is part of an activity ...(I, sec. 23)

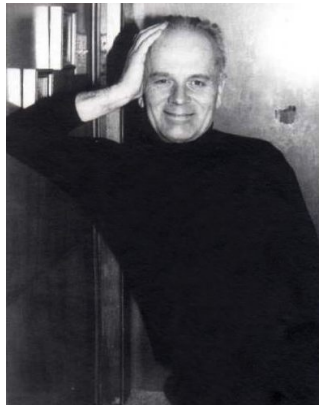
Origins of ATM

Outline

Origins of ATM

Pavel Tichý (1936–October, 26 1994)

- 'Intension in terms of Turing machines' [Tichý, 1969]
  - first explicit formulation of the idea that meanings of expressions are procedures (Turing machines) for computing their denotations



> the fundamental relationship between sentence and procedure is obviously of a semantic nature; namely, the purpose of sentences is to record the outcome of various procedures. Thus e.g. the sentence "The liquid X is an acid" serves to record that the outcome of a definite chemical testing procedure applied to X is positive. (p. 7)

Origins of ATM

Meanings as procedures

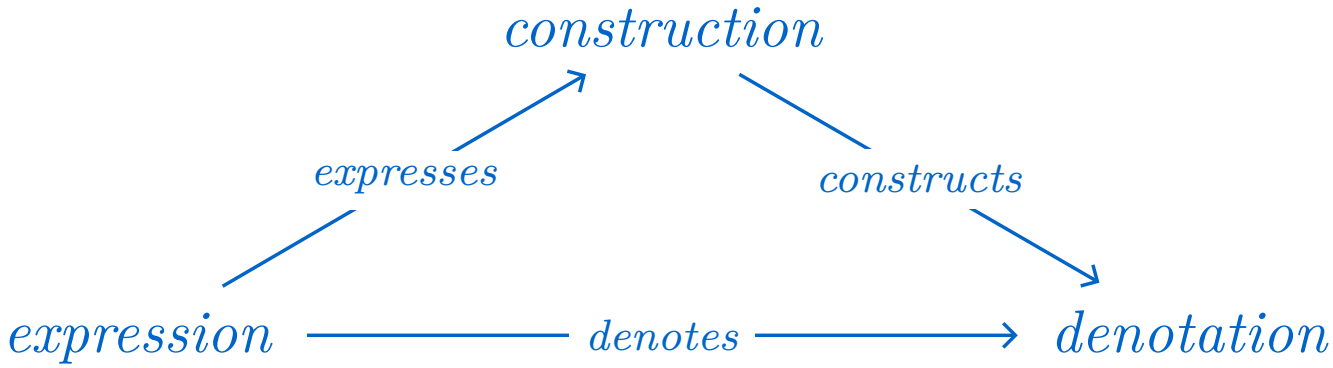
> The sense of an expression is an entity linking the expression with its denotation. For what does it mean to understand, i.e. to know the sense of an expression? It does not mean actually to know its denotation but to know how the denotation can be found, how to pinpoint the denotation of the expression among all the objects of the same type. [...] Thus it seems natural to conceive of concepts as procedures. (pp. 8-9)

- procedures are composed from steps (autonomous/mental and empirical; p. 9)
- only effective mental steps in procedures are allowed (p. 12)

Origins of ATM

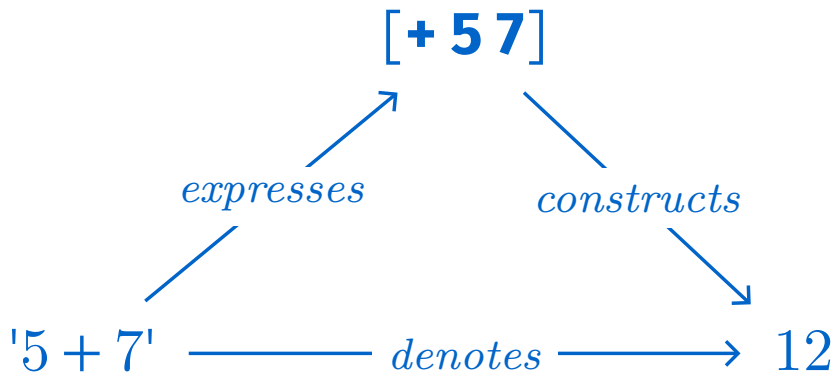
Transparent Intensional Logic (TIL)

- later, Turing machines were generalized into constructions
  - abstract not necessarily effective procedures
- constructions became the basis of his system of Transparent Intensional Logic (TIL) [Tichý, 1988], which is still being actively developed [Duží et al., 2010], [Raclavský et al., 2015])



Origins of ATM

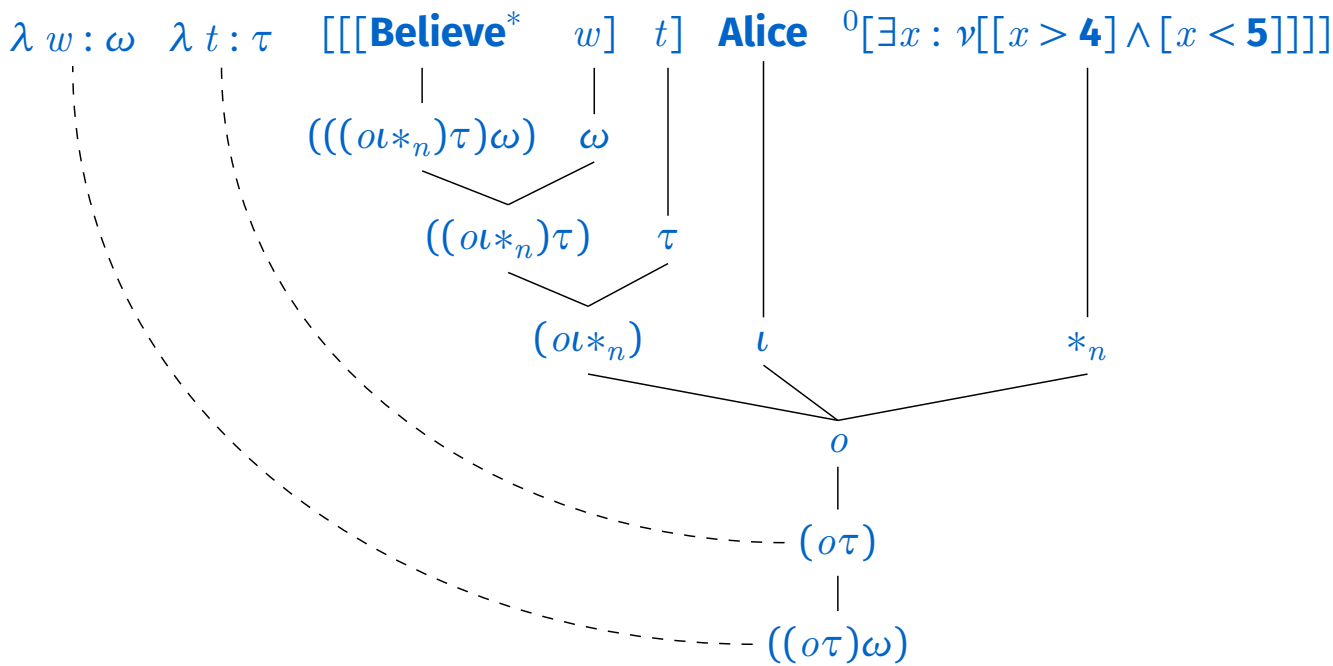
TIL: semantic scheme



Origins of ATM

Semantic analysis and type-checking tree

'Alice believes there is a natural number greater than four but smaller than five.'



Origins of ATM

TIL: constructions

- constructions $\neq$ algorithms, but rather idealized algorithmic computations [Tichý, 1986]

> [N]ot every construction is an algorithmic computation. An algorithmic computation is a sequence of *effective* steps, steps which consist in subjecting a manageable object (usually a symbol or a finite string of symbols) to a feasible operation. A construction, on the other hand, may involve steps which are not of this sort. The application of any function to any argument, for example, counts as a legitimate constructional step; it is not required that the argument be finite or the function effective. [...]. As distinct from an algorithmic computation, a construction is an ideal procedure, not necessarily a mechanical routine for a clerk or a computing machine. (p. 526)

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TIL: summary

Constructions

The key idea is to view meanings as reified not necessarily effective algorithms codifying the procedures for computing denotations of the corresponding natural language expressions.

Meaning

To understand a sentence is to understand a way in which we would go about determining its veridity. Or to use Tichý's terminology, to understand a sentence is to know which particular construction it expresses.

Evolution

meaning $\rightsquigarrow$ Sinn $\rightsquigarrow$ mode of presentation $\rightsquigarrow$ Turing machine $\rightsquigarrow$ TIL construction

Origins of ATM

Crossroads

- Since then the algorithmic approach to natural language meaning attracted a number of researchers from various backgrounds: philosophy, logic, (computational) linguistics, AI, cognitive science, ...
  - see e.g., [Ajdukiewicz, 1965], [Johnson-Laird, 1977], [Woods, 1981], [Hadley, 1989], [Moschovakis, 1993], [Muskens, 2005], [van Lambalgen and Hamm, 2004], [Suppes, 1982], [Van Benthem, 1986], [Szymanik, 2016]
- however by no means it is as widespread or prominent as standard approaches based on model-theoretic, truth-conditional semantics

Origins of ATM

Constructions: a different take

- Parallel to these developments, the notion of construction found its use in another tradition as well: intuitionistic/constructive logic and mathematics ([Brouwer, 1907], [Brouwer, 1907])
- Brouwer-Heyting-Kolmogorov (BHK) semantics, Curry-Howard correspondence
- (intuitionistic) construction = proof, more specifically proof object
- propositions as types and proofs as programs
  - compare: propositions as algorithms
- ATM for formal languages

Origins of ATM

ATM: constructivist tradition

- algorithmic theories of meaning for formal languages have a much stronger tradition
- most notably Martin-Löf's intuitionistic/constructive type theory (CTT, [Martin-Löf, 1984]) and systems in its tradition used for formalizing constructive logic, mathematics and computer programs specifications
- their application to semantics of natural language dealing with empirical discourse is, however, still rather a peripheral area of research ([Sundholm, 1986], [Ranta, 1994])

Further developments

Outline

Further developments

- TIL: automated reasoning ([Duží and Horák, 2019]), machine learning ([Menšík et al., 2019]), natural deduction for partial type logic ([Raclavský et al., 2015])
- CTT: type theory with records ([Cooper, 2005]), modern type theory ([Luo, 2012], [Chatzikyriakidis and Luo, 2013])
  - offshoots: proof-theoretic semantics [Francez, 2015], meaning via elimination/reduction rules [De Queiroz, 2008], [Oliveira, 2019]

Challenges

Outline

Challenges

Loose/strict notions of procedure

- Generally, the closer we get to the natural language the looser conceptions we get:
- formal languages
  - TIL: constructions with only autonomous steps
  - CTT: proof objects as effective algorithms
- natural languages
  - TIL: constructions with autonomous + empirical steps
  - CTT: proof objects as effective algorithms + proof objects as truth-makers (empirical objects, state of affairs, observations, ...)

> I will take procedural processes to include not only symbolic computations, but also the kinds of analog processes exemplified in connectionist (PDP) nets and sense transducers. That is, if a mechanism consistently "computes a function" (or does so to a high degree of approximation) it will, for our purposes, be taken to "execute a procedure." ([Hadley, 1989], p. 144)

Challenges

Atomic empirical sentences

- Closely related to the previous issue
  - How to explain in a procedural terms the meaning of atomic empirical sentences?
    - assume that meanings of atomic sentences are given outside the system
    - include procedures for determining meanings of even atomic sentences

> I cannot pretend that all problems that such a theory [CTT] would have to overcome, say, concerning ordinary empirical discourse, have been resolved. In that area an act of demonstration will be an act of perception, and the verification-object will be the object of the act of perception, namely, what is perceived in the act in question. [...] In this formulation c *is a proof of* A is replaced by (the state of affairs) s *is a truth-maker for* A, or s makes A true. [...] The verification-object serves as the truth-maker for the proposition. ([Sundholm, 2000] pp. 208--209)

Conclusion

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Conclusion

Summary

- first explicit development of ATM for natural language carried out by Pavel Tichý in late 60s
  - theory of non-constructive constructions
- alternative take on constructions and natural language semantics brought constructive type theory of Martin-Löf
  - theory of constructive constructions
- ATM has proponents across many disciplines but never really achieved mainstream status (with the exception of non-empirical discourse)
- partly probably due to the fluctuating notion of algorithm

Conclusion

Thank you for your attention.

Conclusion

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